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XCCurv: the 2D modeller User's Guide - Version 4.0

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Abstract

This report describes the *XCCurv* package. This is a program for modelling free form 2D curves, which is only based on NURBS (Non Uniform Rational B-Splines) mathematics primitive.

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Chapter 1

What is *XCCurv*?

XCCurv is the 2D modeller of the *XCModel* system [XCMODEL00]. This program is self-contained and executable from the *xcmodel* console window. It is distributed with the archive **xccurvdev.tar.gz** (development version) and **xccurvusr.tar.gz** (executable version). Downloading and installation instructions are in [XCMODEL00].
XCCurv is a system based on NURBS mathematics primitive, for modelling 2D curves. This interactive graphics system was designed bearing in mind two main objectives:

- to provide a development environment for the experimentation of new techniques and algorithms in the sector of geometric modelling;
- to provide a learning environment and a practical application of the theories of geometric modelling presented in many books and papers (see references).

For these reasons, *XCCurv* is very different from other 2D CAD systems on the market. Apart from being based on NURBS, it has general characteristics, not belonging to CAD systems, allowing the user to provide all the data needed to define a curve (such as knot partition, weights, type of parametrization), while CAD systems remain more restrictive, imposing methods that are considered efficient, but are invariable.
For example, *XCCurv* allows the user to choose the NURBS degree, and this degree can have up to a maximum value of MAXG (20 in this release). In CAD systems, much lower degrees of spline are used (at most cubic).
As a result, the algorithms implemented in *XCCurv* are as general as possible, in order to be applied in all the cases that the user may require. This could limit the efficiency of the system. For example, the calculation of the B-spline functions (the basis of the space of the polynomial spline) exploits the well-known recurrence formula due to C. de Boor [DEB78]. In CAD systems, on the other hand, algorithms are used that make the calculation of the B-splines, as long as they are of low degree, through the conversion

of each span into the power basis and the use of the Horner method.
Additionally, while in a CAD system 'circle curve' or 'circle arc curve', etc., are defined as being primitive, *XCCurv* provides the instruments for the creation of these curves, while waiting for the users to have the basic knowledge that will allow them to be constructed.
The code is written in ANSI C language and can be executed on various types of workstation or PC running a Unix operating system with Xwindow. In order to make the system portable, it was decided to use a Graphic User Interface made with *xtools* library [XTOOLS00], provided with this package.

Chapter 2

How to work with *XCCurv*

When *XCCurv* is set up, it opens the window shown in fig.2.1. The Main

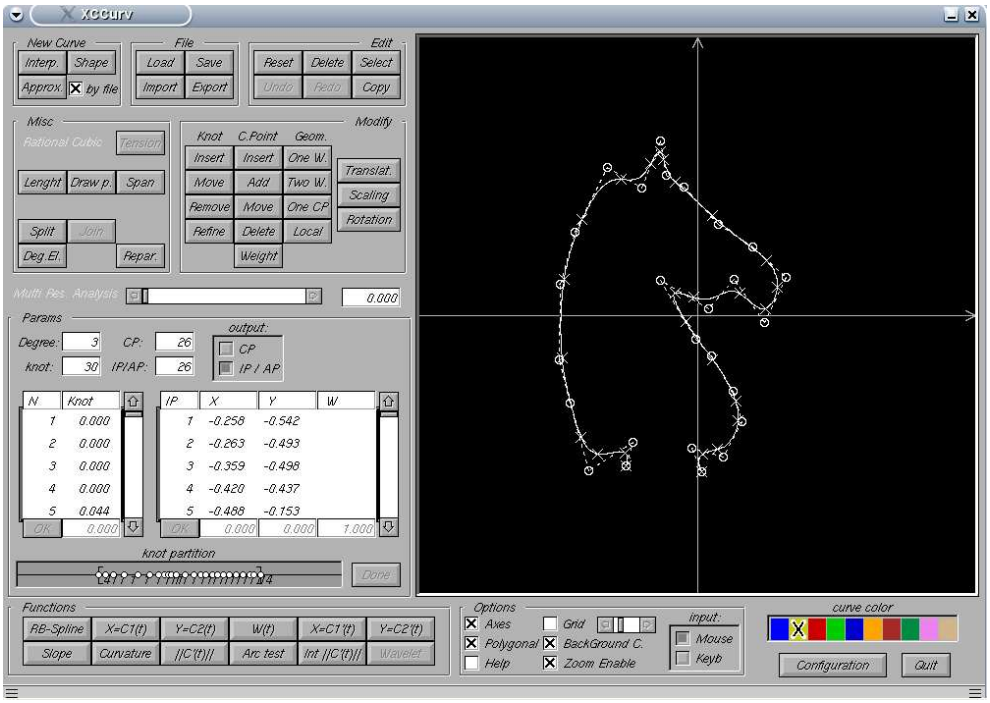

Figure 2.1: Main Window

Window is partitioned in sectors that we have named:

- **New Curve** creates a curve from scrach; this creation step starts by control points, or by interpolation/approximation points given interactively or loading a file;

- **File** to load/save a curve or its points and to import/export a set of curves in IGES format;
- **Edit** includes some useful function such as select a curve, delete a curve, etc.;
- **Misc** consists of a miscellanea collection of functions;
- **Modify** contains the functions available to modify the active NURBS curve;
- **Params** is a large sector of the Main Window containing the definition parameters of the active curve;
- **Functions** consists of 12 buttons corresponding to 12 different function graphs relative to the active curve;
- **Options** allows the user to set some functions of *XCCurv*;
- **Curve Color box** allows the user to choose the color to represent a non active curve;
- **Configuration and Quit bottons** allows the user to configure *XCCurv*, save this configuration for a next work session and quit *XCCurv*.

As soon as this window appears, you will immediately see which actions are possible. *XCCurv* provides help messages viewed along the bottom of the Main Window whenever the cursor moves over buttons, bars, text-boxes, etc. Note that there is also an on-line user's guide, that can be abled by the check-box *Help*, and that guides the user, step by step, in his/her actions.
As in any interactive graphics system, *XCCurv* uses the keyboard and the mouse. In this text we use the expression "click on something with the LMB" (Left Mouse Button) or "the CMB" (Centre Mouse Button) or "the RMB" (Right Mouse Button), when the user places the mouse pointer on something on the screen and presses and releases (to click) a mouse button. This manual is organised to explain all the functionalities and how *XCCurv* works.

2.1 What to do with *XCCurv*

XCCurv is used to model and examine traditional and free-form 2D curves. It is possible to work simultaneously on several curves. The selected (active) curve is visualized in white and it can be modelled or examined. The non-active curves are shown in the selected color from the Curve Color menu in which they were created. There are several techniques for creating curves, from interactive to automatic by interpolation and approximation methods

(see **New Curve** sector). *XCCurv* allows the user to examine the active curve parameters and some useful test functions, such as slope, curvature, etc. (see **Functions** sector). *XCCurv* allows the user to apply and analyze some modelling tools, such as knot-insertion, knot-removal, knot-refinement, etc. (see **Modify** sector). The active curve can be modified by working directly on its parameters or aided by the system giving some geometric constraints. The active curve can be modified by geometric transformation, such as translation, scaling and rotation. *XCCurv* allows the user to save the active curve in a file (.db extension), such as save its control points and interpolation/approximation points (.cp and .ip extensions) and to load a curve or other file previously saved. It is also possible to import and export the modelled curves in the standard IGES format (see **File** sector). In this release there are a lot of new functions and greater graphics interaction. Of particular interest we underline the Multiresolution Analysis option, implemented in a prototypal version, that uses a multiresolution curve representation.

Chapter 3

Params

In the Params sector, the numeric parameters that define the active curve, are permanently shown and up-date; namely, the degree, the number of knots in the extended partition, the number of control points and, if the curve was generated by interpolating or approximating data points, the number of these informations. Two tables show the knot values, the control points coordinates within their weights or, in alternative, the interpolation/approximation point coordinates. It is also graphically represented the knot partition. These information are not only useful to be inspected (as output parameters), but overall to be modified and then change the shape of the curve (as input parameters); this is the case when the input has been chosen to be given by keyboard; when it is the text-boxes under the tables, the OK and Done buttons are able.

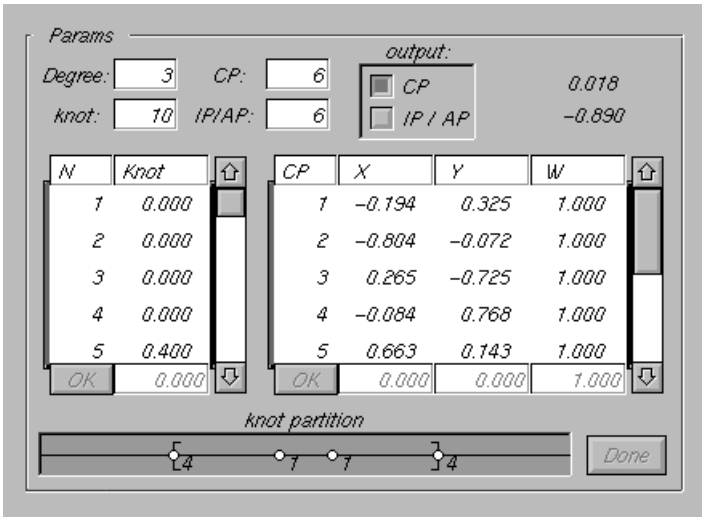

Figure 3.1: Params Area

Chapter 4

Curve Color, Configuration and Quit

4.1 Curve Color menu

By clicking on a color with LMB this menu allows us to set the color for the curve that will be modelled; more precisely this will be the color of the curve when it will be no more the active curve. In fact we remember that when a curve is active its color is white. If the user does not set a color, the system automatically uses the next color in the menu for the curve.

4.2 Configuration button

This button opens the following Configuration Window (see fig.4.1) from which is possible to change the resolution or better to choose the resolution used from the graphics display so that *XCCurv* can be properly used. In

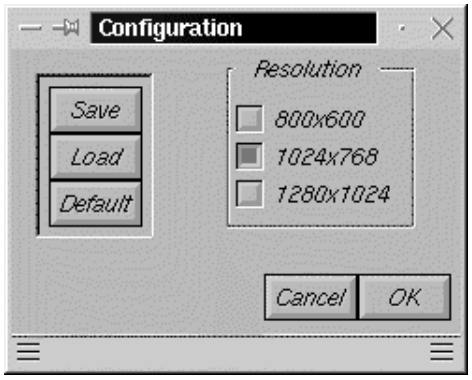

Figure 4.1: Configuration Window

this window are able the Load and Default buttons that allow the user

respectively to set the state of each check-point, check-box, function window dimensions and the values in all the text-boxes of *XCCurv* by loading a configuration file or by some default values. It is also possible to save the current state of all these objects in a configuration file using the Save button.

4.3 Quit button

This button allows the user to quit *XCCurv*.

Chapter 5

Options

This sector allows the user to choose some functions of *XCCurv*. The checkboxes indicate the selection of a yes/no option.

Figure 5.1: Option Area

5.1 Axes

This option ables/unables the drawing of the coordinate axes in the *Curves* window.

5.2 Polygonal

This option ables/unables the viewing of the control polygon of the modelled curves.

5.3 Grid

This option allows the user to able/unable a reference grid in the *Curves* window. If this is abled, every operation following it will be carried out in a system of discrete coordinates, that is, every point is approximated to the nearest point on the grid.
The linked bar, if the grid is active, allows it to be doubled or halved obtaining a grid finer or coarser.

5.4 BackGround Curves

This option ables/unables the viewing of all the non-active curves. This is useful to better view the active curve and to speed-up the interactive operation on the active curve.

5.5 Zoom Enable

This check-box allows us the following facilities:

- Zoom In
- Zoom Out
- Reset

Zoom In and **Zoom Out** respectively allow you to magnify and reduce the image curves in the *Curves* window and are actived respectively by the LMB and RMB clicking on the center of the region to magnify or reduce. To reset the *Curves* window to $[-1, 1] \times [-1, 1]$ use the CMB.

5.6 Help

This check-box is used to able/unable the on-line help of *XCCurv*. For every selected operation, this help guides the user on how to carry out the operation correctly.

Figure 5.2: Help Window

5.7 Input selector

The **Mouse** and **Keyboard** check-boxes are mutually exclusive. If **Mouse** is chosen, almost all the operations for data insertion will be carried out using the mouse, whereas, by choosing **Keyboard**, these will be made using the keyboard.

Chapter 6

File

These buttons (see the Main Window in fig. 2.1) allow the user to to read/write from/on the data files for the NURBS curves.

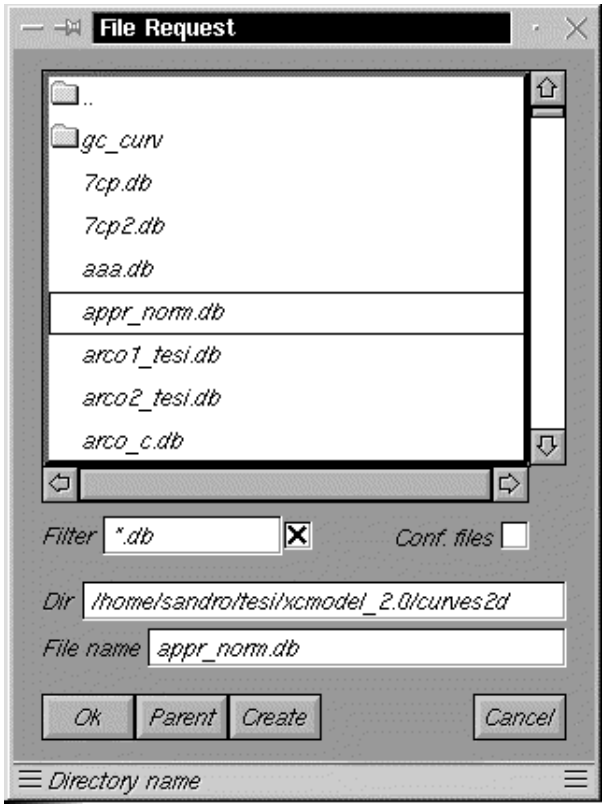

Figure 6.1: File Request Window

6.1 Load button

This button allows a file, with the extension .db, to be loaded. The files with this extension contain all the data needed to define a 2D NURBS curve in a standard format (see Data file formats section). The curve defined by this type of data file will be visualized and will be the active curve.

6.2 Save button

This button allows all the definition data for a 2D NURBS curve active in a file, with a .db extension, to be saved (see db format).

If the active curve has been created by a process of interpolation or approximation of data, this button allows the interpolation or approximation points used to be saved in a file with an .ip extension.

It is also possible to save the control points of the active curve in a file with the .cp extension.

6.3 Import button

This button allows the user to load a file in IGES standard format (see Data file format section) containing a set of NURBS 2D curves (IGES entity no.126).

6.4 Export button

This button allows the user to save a set of curves, selected with LMB from the modelled curves, or all the curves clicking CMB, in a file in IGES standard format (see Data file format section).

Chapter 7

New Curve

This sector presents the three different curve creation modes provided by *XCCurv*:

- Shape approximation from a set of 2D points (Control Points (CP));
- Interpolation from a set of 2D points (Interpolation Points (IP));
- Norm Approximation from a set of 2D points (Approximation Points (AP))

For each of these modes, the user can input the 2D points using the mouse or the keyboard. In this latter case the coordinate values must be digitized in the properly fields in the Params sector. If the check-box labelled "by file" is setted at on , these points can be loaded from a data file.

7.1 Shape Approximation

After the input of the Control Points, the window of fig.7.1 is shown where the user must:

1. set the degree of the curve: digit the degree in the text-box;
2. choose extended partition Δ^*. The following choices are able:
   - Equally spaced knots
   - Uniform knots
   - Periodic knots
   - Manual knots
   - Chord lenght parametrization knots

We describe the possible choices for extended partitions; in any case the knots t_m and t_{K+m+1} are set to the end interval domain points, that is to 0 and 1.

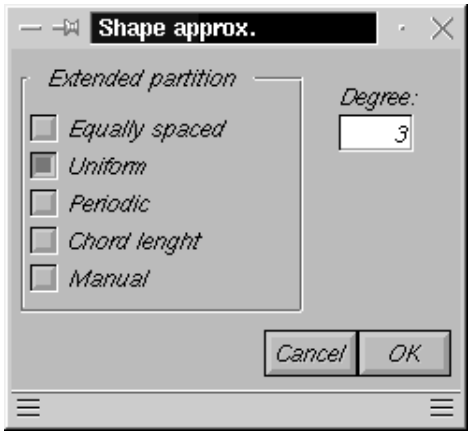

Figure 7.1: Shape Approximation Window

- **Equally spaced and Periodic knots**
  - interior knots are set equally spaced in the [0,1] parametric domain. Let K be the number of interior knots (it holds $K = ncp - m$ where ncp is the CP number and m is the curve order), then t_i are given by:

$$t_i = \frac{i-m}{K+1} \qquad i = m+1, \ldots, K+m$$

  - exterior knots are defined by

$$\begin{aligned} t_i &= a - (b - t_{i+K+1}) \quad i = 1, \ldots, m-1 \\ t_i &= b + (t_{i-K-1} - a) \quad i = K+m+2, \ldots, K+2m \end{aligned}$$

  where $[a, b]$ is the parametric domain ($[0, 1]$ in this release). Since interior knots are equally spaced, we have the same extended partition both in Equally spaced and Periodic selections.

- **Uniform knots**
  - interior knots are equally spaced;
  - exterior knots are coincident with the end interval points:

$$\begin{aligned} t_i &= 0.0 \quad i = 1, \ldots, m-1 \\ t_i &= 1.0 \quad i = K+m+2, \ldots, 2m+K \end{aligned}$$

- **Manual knots**
  The extended partition can be setted by the user. With the mouse he/she can set all the knots, except for t_m and $tK + m + 1$ that are automatically set to be coincident with the end interval points; with the keyboard he/she can set all the knots.

  - Mouse → click $K + 2m - 2$ times on the interval domain with LMB (*knot partition* window)
  - Keyb → digit all knot values, that is $K + 2m$ values, in the textbox.

- **Chord length parametrization knots**
  - interior knots are chosen so that:

$$\frac{t_{i+1} - t_i}{t_{i+2} - t_{i+1}} = \frac{\sum_{j=i-m+1}^{i-1} \|P_j - P_{j-1}\|_2}{\sum_{j=i-m+2}^{i} \|P_j - P_{j-1}\|_2}$$

    where P_j are the curve CPs.
  - exterior knots are coincident with the end interval points:

  Note that all automatic extended partitions have interior knots with a multiplicity to 1. It is only possible to set knots manually with a multiplicity greater than 1. In this case to elevate the multiplicity of a knot click on it with CMB.

The weights are set automatically to 1. To change their values, you must use the Modify Weight button in the Options sector. To summarise, once we have given all the following information:

- control points (CPs);
- degree g;
- knot partition (Δ^*);
- weights (optionally);

XCCurv can proceed to compute and graphically represent the curve.

7.2 Interpolation button and IP file

This button provides the creation of an interpolation curve starting from some given 2D points (IP). After these points have been given, the window of fig.7.2 is shown, and the user must:

1. choose an interpolation Method:
   - Lagrange interpolation
   - Hermite interpolation
   - Periodic interpolation

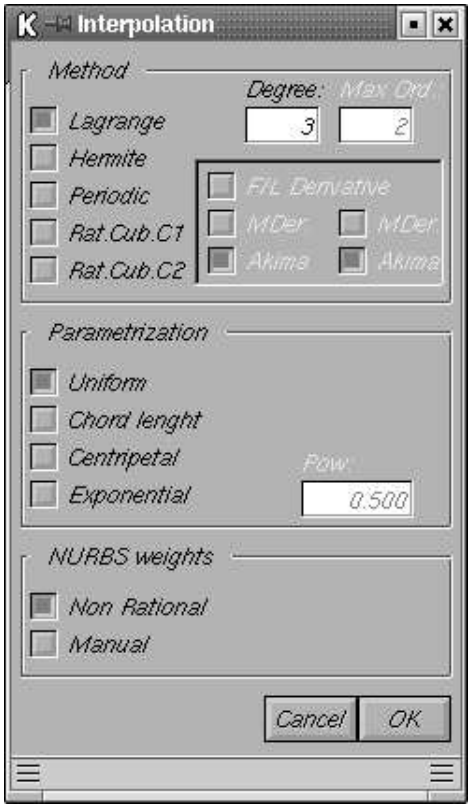

Figure 7.2: Interpolation Window

- Rational cubic interpolation C^1 with tension
- Rational cubic interpolation C^2 with tension

2. choose a Parametrization strategy:

   - Uniform
   - Chord length
   - Centripetal
   - Exponential

3. Choose NURBS weights:

   - Non rational
   - Manual

We describe the available interpolation methods:

- **Lagrange interpolation**
  Let $\{Q_i\}$ $i = 1, \ldots, n$, be the interpolation points. This method computes the NURBS curve $\underline{C}(t)$, so that:

  $$\underline{C}(\tau_i) = Q_i \qquad i = 1, \ldots, n$$

  The degree of the curve must be given. The knot partition will be computed automatically so that the Schoenberg-Whitney conditions are satisfied. Exterior knots are coincident with the end interval points.

- **Hermite interpolation**
  Let $\{Q_i^\ell\}$ $i = 1, \ldots, n$, and $\ell = 0, \ldots, \ell_i$ be the interpolation points. This method computes the NURBS curve $\underline{C}(t)$ so that:

  $$\underline{C}^\ell(\tau_i) = Q_i^\ell \quad i = 1, \ldots, n \;\; \ell = 0, \ldots, \ell_i$$

  This option allows the following choices:

  - **F/L Derivative**
    This method, in addition to the interpolation points $\{Q_i^0\}$, $i = 1, \ldots, n$, interpolates the first derivative at the end interval points; that is Q_1^1 and Q_n^1 ($\ell_1 = \ell_n = 1$ and $\ell_i = 0$ for $i = 2, \ldots, n-1$) must be given. The interpolation curve is of degree 3. The interior knots are chosen to coincide with the interpolation parameter values τ_i; the exterior knots are coincident with the end interval points.
  - **Manual Derivative**
    In addition to the interpolation points $\{Q_i^0\}$, $i = 1, \ldots, n$, the Max. Order $d \in \{1, 2, 3, 4\}$ must be given and then the information $\{Q_i^\ell\}$, $i = 1, \ldots, n$, and $\ell = 1, \ldots, d$. The interpolation curve is of degree $2d+1$. The interior knots are chosen to coincide with the interpolation parameter values τ_i and are of multiplicity $d+1$. The exterior knots are coincident with the end interval points.
  - **Akima**
    Start from the given points $\{Q_i^0\}$, $i = 1, \ldots, n$. This option computes, the points $\{Q_i^1\}$, $i = 1, \ldots, n$ with Akima technique. Then an interpolation of all this information, with a curve of degree 3, is performed. The interior knots are chosen to coincide with the interpolation parameter values τ_i and have multiplicity 2. The exterior knots are coincident with the end interval points.

- **Periodic interpolation**
  Let $\{Q_i\}_{i=1,\ldots,n}$, be the interpolation points. This method computes

the NURBS curve $\underline{C}(t)$ so that:

$$\begin{cases} \underline{C}(\tau_i) = Q_i \qquad i = 0, \ldots, n \\ \underline{C}^1(\tau_1) = \underline{C}^1(\tau_n) \\ \underline{C}^2(\tau_1) = \underline{C}^2(\tau_n) \end{cases}$$

The interior knots are chosen to coincide with the interpolation parameter values $i\tau_i$. The exterior knots are chosen to satisfy the periodic conditions automatically.

- **Rational cubic C^1 interpolation**
  Let $\{Q_i^\ell\}_{i=1,\ldots,n}$, and $\ell = 0, 1$ be the interpolation points and w_i $i = 1, \ldots, n-1$, be tension parameters associated with each pair $\{Q_i^\ell\}$ $\{Q_{i+1}^\ell\}$ of points. This option allows the following choices:
  - Manual Derivative
  - Akima

  that are the same already seen in the Hermite interpolation case. This method computes the interpolation NURBS curve (cubic over quadratic) with the property of being globally C^1 and for $w_i \to \infty$ $\forall i$ to converge to the polygonal defined by the interpolation points. If $w_i = 1 \quad \forall i$, it is the cubic Hermite interpolation curve.

- **Rational cubic C^2 interpolation**
  Let $\{Q_i^\ell\}_{i=1,\ldots,n}$, and $\ell = 0, \cdots, \ell_i$ where $\ell_1 = \ell_n = 1$ and $\ell_i = 0 \quad i = 2, \cdots, n-1$ be the interpolation points and w_i $i = 1, \ldots, n-1$, be tension parameters associated with each pair $\{Q_i^\ell\}$ $\{Q_{i+1}^\ell\}$ of points. This method computes the interpolation NURBS curve (cubic over quadratic) with the property of being globally C^2. If $w_i = 1 \quad \forall i$, it is the cubic Hermite interpolation curve that we called F/L Derivative.

After the choice of an interpolation method, the user must choose the parametrization strategy, that is, the rule to set the interpolation parameter values τ_i. *XCCurv* provides four strategies:

- **Chord length parametrization**
  Sets the τ_i parameters in the interval $[0, 1]$, so that:

$$\frac{\tau_{i+1} - \tau_i}{\tau_{i+2} - \tau_{i+1}} = \frac{\|Q_{i+1} - Q_i\|_2}{\|Q_{i+2} - Q_{i+1}\|_2}$$

- **Uniform parametrization**
  Sets the τ_i parameters equally spaced in the interval $[0, 1]$.

- **Centripetal parametrization**
  Sets the τ_i parameters in the interval $[0, 1]$, so that:

$$\frac{\tau_{i+1} - \tau_i}{\tau_{i+2} - \tau_{i+1}} = \left(\frac{\|Q_{i+1} - Q_i\|_2}{\|Q_{i+2} - Q_{i+1}\|_2} \right)^{1/2}$$

- **Exponential parametrization**
  Sets the τ_i parameters in the interval $[0, 1]$, so that:

$$\frac{\tau_{i+1} - \tau_i}{\tau_{i+2} - \tau_{i+1}} = \left(\frac{\|Q_{i+1} - Q_i\|_2}{\|Q_{i+2} - Q_{i+1}\|_2} \right)^{\alpha} \quad \text{with} \quad \alpha > 0$$

Finally, the user must set the NURBS weights to define the NURBS space.

- **Non Rational**
  Each weight is set to 1. Thus the NURBS space is a non-rational spline space.

- **Manual**
  This input can be performed with the keyboard only. For each weight digit its value in the properly text-box.

  Remember that the same number of weights must be set as the number of the interpolation points.

7.2.1 Norm Approximation button and AP file

This button allows the computation of a least square weighted approximation curve starting from some given 2D points (AP). After these points have been given, the window of fig.7.3 is shown, and the user must:

1. choose the values to weight the approximation points:
   - Equal
   - Manual
2. choose an approximation Method:
   - Least square
   - Constrained least square
   - Periodic least square
3. choose curve degree;
4. choose a Parametrization strategy:
   - Uniform

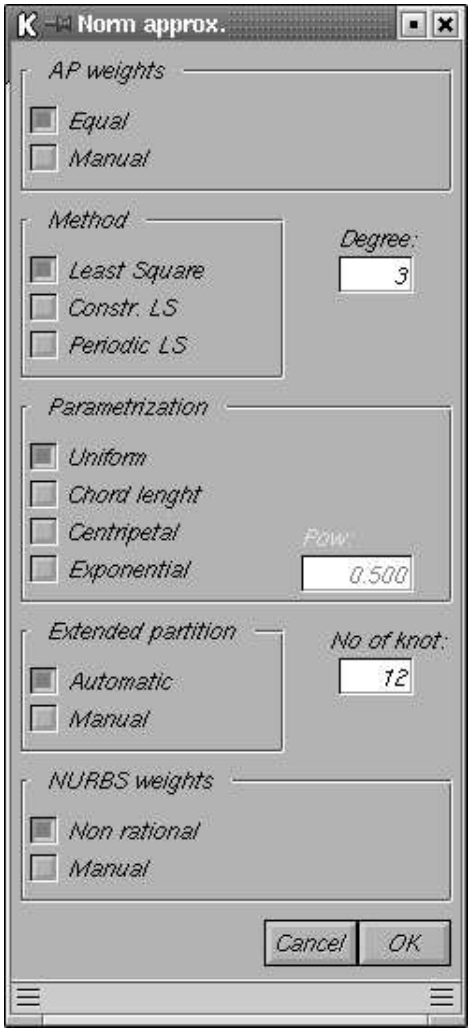

Figure 7.3: Norm Approximation Window

- Chord length
- Centripetal
- Exponential

5. choose Extended partition:
   - Automatic
   - Manual
6. choose the number of knots;
7. choose NURBS weights:

- Non rational
- Manual

The weights to associate to the approximation points can be:

- **Equal**
  This option sets all the weights to 1; non-weighted approximation.

- **Manual**
  This option allows to input the weights by keyboard. In the AP table, in the Params sector, the x, y coordinate points are shown within all the weights to 1; click with LMB the weight to be changed and give the new value in the properly text-box; OK to confirm and Done to quit the input step.

Approximation methods:

- **Least square approximation method**
  Let $\{Q_i\}_{i=1,\dots,n}$, be the approximation points. This method computes the NURBS curve $\underline{C}(t)$, so that the following expression is the minimum value:

$$\sum_{i=1}^{n} \|\underline{C}(\tau_i) - Q_i\|_2^2 = \sum_{i=1}^{n}((C_1(\tau_i) - x_i)^2 + (C_2(\tau_i) - y_i)^2)$$

- **Constrained least square approximation method**
  In addition to satisfying the least square approximation, this method constrains the curve to pass from the first and last given points, that is:

$$\underline{C}(\tau_1) = Q_1 \quad e \quad \underline{C}(\tau_n) = Q_n$$

- **Periodic least square approximation method**
  In addition to satisfying the least square approximation, this method constrains the curve to be periodic, that is:

$$\begin{cases} \underline{C}(\tau_1) = \underline{C}(\tau_n) \\ \underline{C'}(\tau_1) = \underline{C'}(\tau_n) \\ \vdots \\ \underline{C}^{(m-2)}(\tau_1) = \underline{C}^{(m-2)}(\tau_n) \end{cases}$$

Then the user must give the degree curve and choose the parametrization strategy, that is, the rule to set the interpolation parameter values τ_i. *XC-Curv* provides four strategies:

- **Chord length parametrization**
  This option sets the τ_i parameters in the interval $[0, 1]$, so that:
  $$\frac{\tau_{i+1} - \tau_i}{\tau_{i+2} - \tau_{i+1}} = \frac{\|Q_{i+1} - Q_i\|_2}{\|Q_{i+2} - Q_{i+1}\|_2}$$

- **Uniform parametrization**
  This option sets the τ_i parameters equally spaced within the interval $[0, 1]$.

- **Centripetal parametrization**
  This option sets the τ_i parameters in the interval $[0, 1]$, so that:
  $$\frac{\tau_{i+1} - \tau_i}{\tau_{i+2} - \tau_{i+1}} = \left(\frac{\|Q_{i+1} - Q_i\|_2}{\|Q_{i+2} - Q_{i+1}\|_2}\right)^{1/2}$$

- **Exponential parametrization**
  This option sets the τ_i parameters in the interval $[0, 1]$, so that:
  $$\frac{\tau_{i+1} - \tau_i}{\tau_{i+2} - \tau_{i+1}} = \left(\frac{\|Q_{i+1} - Q_i\|_2}{\|Q_{i+2} - Q_{i+1}\|_2}\right)^{\alpha} \quad \text{with} \quad \alpha > 0$$

The next steps are the choice of the knot partition strategy: The latter can be:

- **Manual**
  The user can set the position of the interior knots by the mouse or the keyboard; in the mouse case to quit the input click with RMB while in the keyboard case press the Done button.

- **Automatic**
  In this case the number of knots must be given; *XCCurv* sets the interior knots so that the least square approximation curve will be unique. The exterior knots are set to coincide with the end interval points.

The final step is to define the NURBS space by giving the NURBS weights:

- **Non Rational**
  Each weight is set to 1. Thus the NURBS space is a non-rational spline space.

- **Manual**
  This input can be performed with the keyboard only; digit its value in the properly text-box.

  Remember that the same number of weights must be set as the number of the approximation points.

Chapter 8

Edit

This sector allows the user to manage the curve environment by classic functions as:

8.1 Reset

This button resets *XCCurv*. If the curves have not been saved they will be lost.

8.2 Delete

This option allows the user to delete the active curve. After this operation, no curve will be active. Proceed immediatly to select a curve to be active or create a new curve.

8.3 Select

XCCurv allows the user to model up to MAXC curves at the same time (100 in this release). One of these must be selected as active (appearing in white). All the operations that *XCCurv* provides are only able for the active curve. To select a curve as being active, click on one CP of a curve with LMB and click with RMB to confirm. Note that the last curve created or loaded by a file is set active.

8.4 Copy

This option allows the user to duplicate the active curve. The two curves are one over the other and only moving the last can be appreciated the duplication effect.

8.5 UnDo

This option allows the user to return to the curve situation previous up to the latest 16 modifications.

8.6 ReDo

This option allows the user to repeat the latest modifications eliminated with UnDo.

Chapter 9

Misc

This sector presents a miscellanea collection of useful modelling functions.

9.1 Rational Cubic Tension

This button refers to the two methods of interpolation called Rational Cubic C^1 and Rational Cubic C^2. For a curve created with these interpolation methods this button is abled and provides to change the length of the curve mantaining the interpolation. This is realized by using some tension parameters with local or global effect. To change the tension of the curve between two interpolation points, click with CMB on these two points. Press LMB and drag on the right or on the left respectively to increase or decrease the tension value.

9.2 Curve Lenght

Computes the length of the active curve.

9.3 Draw Points

XCCurv graphically represents the active curve using dots. These dots correspond to curve points with equally spaced parameter values.
This option is useful to test curve parametrization. In fact, if the curve is well-parametrized, these curve points will be equally spaced, even on the curve. To continue and redisplay the curve in the standard mode, click with LMB.

9.4 Span

This button allows the user to visualize and match a knot interval with a curve span.

- **mouse:** to select a knot interval, click on it with LMB; repeat for other knot intervals. RMB to stop.
- **keyb:** to select a knot interval inserts a value belonging to the desired interval or selects directly the left interval knot in the knot list; Done to exit.

9.5 Split

This option allows the user to split the active curve into two curves.

- To split, click on the chosen curve point with LMB.

9.6 Reverse

This button allows for reverse the parametrically definition of the active curve. In other words, the zero parameter value will be now associated to the last curve point and the one parameter value to the first.

9.7 Join

Select the first curve with LMB, then the second curve to join with LMB; the first point of the second curve will be joined to the last point of the first curve.

9.8 Degree Elevation

This button allows the user to make a degree elevation of the curve by 1 degree. This procedure does not modify the curve shape, but only its analytical representation.
To make a degree elevation greater than 1, click on the button more than once.

9.9 Reparametrization

This button opens the following menu, from which it is possible to choose different reparametrization techniques:

- Linear rational
- Span
- $\|C'(t)\|$

- Root arc
- Adaptive span
- Adaptive $\|C'(t)\|$
- Adaptive root arc

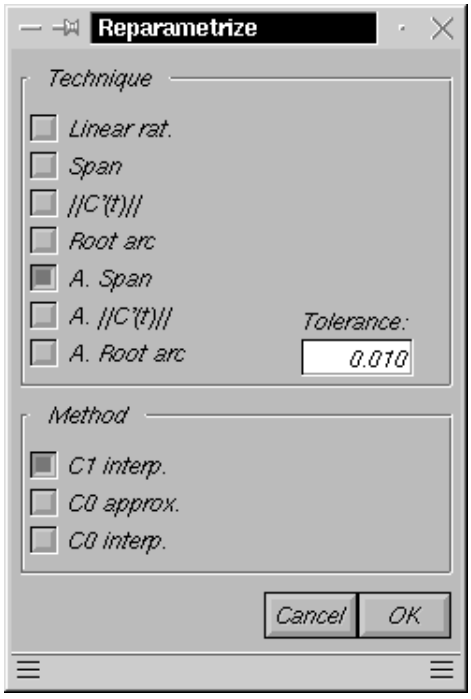

Figure 9.1: Reparametrization Window

These consist in changing the current parameter of the active curve with another parameter using a linear rational reparametrization function that keeps the curve a NURBS too.
All the proposed techniques use a linear rational or a piecewise linear rational function to approximate the arc length parametrization function

$$\varphi(t) = \int_0^t \|C'(u)\|_2 \, du$$

by interpolation or uniform approximation.
For each chosen technique, the following approximation modes are available:

- C^1 interpolation
- C^0 uniform approximation

- C^0 interpolation

The proposed techniques partition the parametric interval in different ways and consequently use a different piecewise reparametrization function.

9.9.1 Linear rational button

This option reparametrizes the whole active curve using a single linear rational function. In this case the choise of C^1 or C^0 interpolation gives the same approximation function for $\varphi(t)$.

9.9.2 Span button

This option performs piecewise reparametrization of the active curve after splitting the parametric domain at the knots.

9.9.3 $\|C'(t)\|$ button

This option performs a piecewise reparametrization of the active curve after splitting the parametric domain at the points where the $\varphi(t)$ function

$$\varphi(t) = \int_0^t \|C'(u)\|_2 \, du$$

changes convexity (concavity).
Because a linear rational function is always convex (concave), this option allows the user to approximate the $\varphi(t)$ shape.

9.9.4 Root arc button

This option performs a piecewise reparametrization of the active curve after splitting the parametric domain at the zeroes of the arc test function $a(t)$

$$a(t) = \frac{\int_0^t \|C'(u)\| \, du}{L} - t \quad t \in [0,1]$$

where L is the curve length ($L = \int_0^1 \|C'(u)\| \, du$).
A well-parametrized curve, in the arc length sense, results in $a(t) = 0 \ \ \forall t$. Splitting the reparametrization at the $a(t)$ zeroes means reparametrizing the curve only where necessary.

The following three techniques, called Adaptive, approximate the arc length parametrization function $\varphi(t)$ adaptively, up to a given tolerance.
The first step is to compute an approximate linear rational function over the whole parametric domain. If tolerance is not reached, the parametric interval is divided into two intervals, and so on:

9.9.5 Adaptive span button

This option divides the interval in correspondence with a knot. Otherwise, it splits the interval in half, if it does not contain knots.

9.9.6 Adaptive $\|C'(t)\|$ button

This option divides the interval into two intervals to respect the concave/convex behaviours of the $\|C'(t)\|$ function. Otherwise, it splits the interval in half, if the $\|C'(t)\|$ function is already concave or convex, but tolerance has not been reached.

9.9.7 Adaptive root arc button

This option divides the interval into two intervals to respect the roots of the $a(t)$ function. Otherwise, it splits the interval in half, if it does not contain other $a(t)$ roots.

9.10 Multi Resolution Analysis

This is an interesting facility of *XCCurv*. Its implementation is prototypal and for this reason is abled only for an active curve of degree 3, with uniform knot partition and with $2^k + 3$ control points with k an integer value.

This option uses a multiresolution curve representation, based on wavelets, that conveniently supports a variety of operations: smoothing a curve; editing the overall form of a curve while preserving its details; and so on. This implementation supports continuous level of smoothing.

When the bar and the text-box are abled, they have position/value k (the initial resolution level of the curve); changing this position/value we smooth or refine the curve mantaining its details (see [FINK94]).

Chapter 10

Modify

This sector provides the capability to modify the active NURBS curve by:

- Knot partition
- Control Points
- Geometric constraints
- Geometric Transformations

By **Modify** we mean variation of the curve parameters. Sometimes this involves a shape curve variation, and sometimes only a different analytical representation.

10.1 Knot buttons

10.1.1 Insert button

This button allows the user to make a knot insertion of 1 or more knots in the actual knot partition. This procedure does not modify the shape of the curve, but only its analytical representation.

Mouse → click on chosen position inside
parametric interval with LMB;
→ click CMB on an existing knot to elevate its multeplicity;
→ click with RMB to stop.

Keyb → digit new knot value in the properly text-box;
→ OK to confirm;
→ Done to exit.

10.1.2 Move button

This button allows the user to move one or more interior knots on the actual partition from their initial position to a new one. This procedure modifies the curve shape.

Mouse → press on a knot with LMB, drag to new position, then release;
→ click with RMB to confirm.

Keyb → select a knot by clicking on it with LMB in the knot table;
→ digit its new value (use Backspace to delete the old value);
→ OK to confirm;
→ Done to exit.

10.1.3 Remove button

This option tries to remove a knot from the actual knot partition without modifying the curve shape. If this procedure is successful, the effect will be a curve with a different analytical representation.

Mouse → click on a knot with LMB;
→ click with RMB to confirm.

Keyb → select a knot by clicking on it with LMB in the knot table;
→ OK to confirm;
→ Done to exit.

10.1.4 Refine button

The knot partition wil be refined; in the middle point of each knot interval will be insert a new knot.

10.2 Control Point buttons

These buttons allow the user to modify the control points and weights.

10.2.1 Insert button

This button performs the operation known as Inverse Knot-Insertion. This consists in the specification of a point on the control polygon. *XCCurv* identifies a point on the parametric domain from which, by knot insertion, a new polygonal can be obtained having the chosen point as one of the control

points.
This procedure does not modify the curve shape, but only its analytical representation.

Mouse → click on chosen polygonal point with LMB.

Keyb → digit its coordinates and weight values;
→ OK to confirm each value;
→ Done to exit.

Note that, in the Keyboard case, if the new coordinate point does not belong to the polygonal, a message appear and the user must digit others coordinate points.

10.2.2 Add button

This button allows the user to add new control points. This procedure modifies the curve shape.

Mouse → to select where to add a CP,
click on a polygonal segment with CMB;
→ click on new position with LMB;
→ click with CMB on an existing CP to elevate
its multiplicity;
→ clicking with LMB without a previous segment selection,
inserts a CP as first.

Keyb → to select the position at which to add a CP,
click on CP table with LMB;
→ digit its coordinates and weight values;
→ OK to confirm each value;
→ Done to exit.

10.2.3 Move button

This button allows the user to move a control point. This procedure modifies the control polygon and therefore the curve shape.

Mouse → press on chosen CP with LMB and drag
to the new position;
→ release to exit.

Keyb → click on a CP with LMB in CP table;
→ digit its coordinate values;
→ OK to confirm;
→ Done to exit.

10.2.4 Delete button

This button allows the user to delete a control point. This procedure modifies the control polygon and therefore the curve shape.

Mouse → click on chosen CP with LMB.
Keyb → click on a CP with LMB in CP table;
→ OK to confirm;
→ Done to exit.

10.2.5 Weight button

This button allows the user to modify a weight value associated with a control point. This procedure modifies the curve shape.

Mouse → press on chosen CP with LMB and
drag on the right to increase the weight value,
on the left to decrease the weight value;
→ release to exit.

Keyb → click on chosen CP with LMB in CP table;
→ digit new weight value;
→ OK to confirm;
→ Done to exit.

10.3 Geometric buttons

These buttons allow the user to make some geometric modifications to the shape of the curve. By **Geometric modify** we mean that it is possible to impose some geometric constraints, such as, passing over a given point.

10.3.1 One Weight button

This button allows the user to modify the active curve so that it passes over a point S^* by only changing a weight. More precisely, the user must select a curve point (S) and a point on the control polygon (P). Then he/she must choose S^* on the straight segment MP (see fig.10.1), where M is the analog of S, when the weight associated to the control point P is zero.

mouse → click on a curve point (S) with CMB;
→ click on a polygonal point (P) with CMB;

The line defined by S and P and the segment MP on this line will be shown.

→ press with LMB and drag to the chosen S^*;
→ release to exit.

Note that, the dragging stage quit if S^* touches P or M.

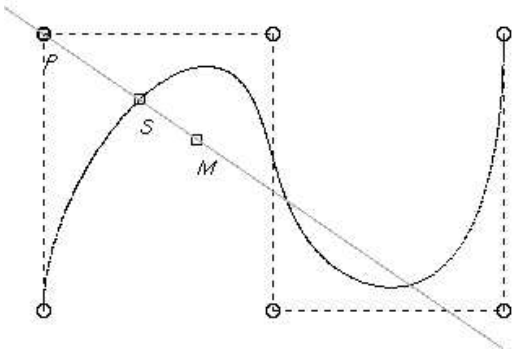

Figure 10.1: One weight modify

10.3.2 Two Weight button

This button allows the user to modify the active curve so that it passes over a point S^* by changing only two weights. More precisely, the user must select two control points P_i and P_j; *XCCurv* shows a curve point S and a point M. S is the curve point that will be moved to S^*, while M represents the curve point S, if the weights associated with the two selected control points were zeroes. Now the user can set S^* to be in the triangle P_iP_jM (see fig. 10.2).
XCCurv also shows the line through S and M. If the user chooses S^* in the triangle, but also on SM, the curve will be modified to be simmetry preserving.

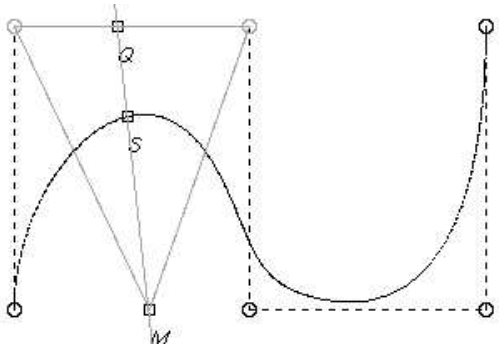

Figure 10.2: Two weights modify

mouse → click on a CP (P_i) with CMB;
→ click on a CP (P_j) with CMB;
→ press on a point inside the P_iP_jM triangle with LMB and drag to the chosen S^*;
→ released to exit.

10.3.3 One CP button

This button allows the user to modify the active curve so that it passes over a point S^* by changing only one control point. More precisely, the user must select a curve point (S); *XCCurv* finds the control point P nearest to S and shows the line through S and the control point. The user must set S^* to be on this line.

Mouse → press on a curve point (S) with LMB and drag to the new position;
→ released to exit.

XCCurv shows the line defined by S and P.

10.3.4 Local button

This button allows the user to select a curve segment and modify only the shape of this curve segment only. This procedure consists in local knot refinement and in a control point movement.

Mouse → click on a curve point with CMB to select the first segment end point;
→ click on a curve point with CMB to select the last segment end point;
→ press with LMB and drag to a new position;
→ released to exit.

XCCurv computes the parametric interval associated with this curve segment and inserts $m+1$ knots (with m as the order curve) inside this interval. Now there is a basis function with this interval as support. By moving the control point associated with this basis function, only the curve segment will be modified.

10.4 Transformation buttons

These buttons allow the user to make the following geometric transformations:

10.4.1 Translation button

This button allows the user to translate the active curve from its position to a new one or some its control points only.

Mouse → select with CMB the control points to be translated;
nobody selection means all control points;
→ press with LMB to define the center of translation;
→ drag to define the translation vector;
→ released to exit.

Keyb → digit the translation vector coordinates
in the Window Transformation;
→ OK to confirm

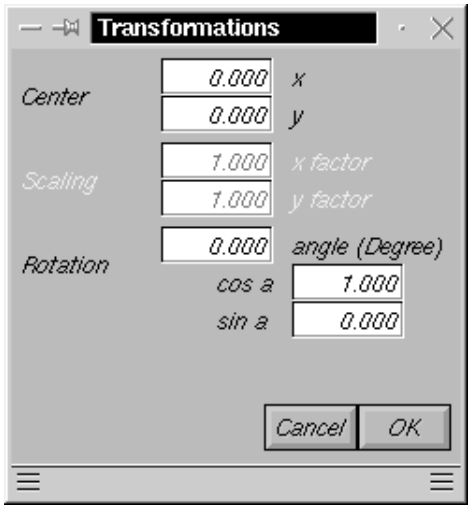

Figure 10.3: Tranformation Window

10.4.2 Scaling button

This button allows the user to scale the active curve.

Mouse → select with CMB the control points to be scaled;
nobody selection means all control points;
→ press with LMB to define the center of scale;
→ drag to define the scaling factors;
→ released to exit.

Keyb → digit the coordinates of the center of scale
in the Window Transformation;
→ digit the scaling factors;
→ OK to confirm

10.4.3 Rotation button

Mouse → select with CMB the control points to be rotated;
nobody selection means all control points;
→ press with LMB to define the center of rotation;
→ drag to define the angle of rotation;
→ released to exit.

Keyb → digit coordinates of the center of rotation
in the Window Transformation;
→ digit the angle in degree;
→ OK to confirm

Chapter 11

Functions buttons

These buttons provide the visualization of some different functions. These are very useful when you need to inspect the analytic and shape characteristics of the active curve.
XCCurv is able to show all these functions contemporarily and to resize each function window to better understand the function details.

11.1 RB-Spline button

This option shows the graphs of the basis function for the NURBS space $R(P_m, M, \Delta, W)$ to which the active curve belongs.

11.2 $X = C_1(t)$ and $Y = C_2(t)$ buttons.

These options show the graphs of the components of the active curve. The control polygons of these functions that have vertices $(\xi_i, x_i)_{i=1,\ldots,m+K}$ (or $(\xi_i, y_i)_{i=1,\ldots,m+K}$) are also shown. The ξ_i are the nodes and (x_i, y_i) are the control point coordinates.
If the active curve has been created by interpolation or approximation, the points (t_i, x_i) (o (t_i, y_i)) where t_i are the parameter values associated to the interpolation or approximation points (x_i, y_i) will be shown.

11.3 $W(t)$ button

This option shows the graph of the weight function, that is the denominator function of the rational active curve.

11.4 $X = C_1'(t)$ and $Y = C_2'(t)$ buttons

These options show the graphs of the first derivative functions of the curve component functions.

11.5 Slope button

This option shows the graph of the slope function $v(t)$, defined by:

$$v(t) = \frac{C_2'(t)}{C_1'(t)}$$

11.6 Curvature button

This option shows the graph of the curvature function $k(t)$, defined by:

$$k(t) = \frac{C_1'(t)C_2''(t) - C_2'(t)C_1''(t)}{([C_1'(t)]^2 + [C_2'(t)]^2)^{3/2}}$$

11.7 $\|C'(t)\|$ button

This option shows the graph of the function $\|C'(t)\|_2 - L$, where L is the curve length.
This graph is useful to show the user whether the active curve is well-parametrized or not. In fact, if a curve is well-parametrized, in the arc length sense, then

$$\|C'(t)\| \longrightarrow L$$

and therefore $\|C'(t)\| - L$ must zero.
In addition, the minimum and maximum function values are shown.

11.8 Arc test button

This option shows the graph of the function $a(t)$, defined by:

$$a(t) = \frac{\int_0^t \|C'(u)\|\, du}{L} - t \quad t \in [0,1],$$

where L is the curve length ($L = \int_0^1 \|C'(u)\|\, du$). This graph is useful to show the user whether the active curve is well-parametrized or not. In fact, if a curve is well-parametrized, in the arc length sense, then $a(t)$ must be zero.
In addition the minimum and maximum function values are shown.

11.9 Int $\|C'(t)\|$ button

This option shows the graph of the function $\varphi(t)$, defined by:

$$\varphi(t) = \int_0^t \|C'(u)\|_2\, du$$

This graph is useful to show the user whether the active curve is well-parametrized or not. In fact, if a curve is well parametrized, in the arc length sense, then $\varphi(t)$ approximates a linear function; $\varphi(t)$ is the analytic reparametrization function. When you choose to reparametrize the active curve, you can compare the $\varphi(t)$ function with the reparametrization function that *XCCurv* uses.

11.10 Wavelet button

This option shows the graphs of the wavelet functions associated to the RB-spline functions; this button is able when the active curve allows us to perform a MultiResolution Analysis.

Chapter 12

Data file formats

In this section the sintax of each file format used by *XCCurv* is given and explained. The data files created or used by *XCCurv* are stored in the directory `xcmodel/curves2d` as default. The # character in the following is a comment to the data in the file.

12.1 NURBS curve 2D

The following example file is `xcmodel/curves2d/c9p.db`. The .db extension identifies a NURBS entity.

```
FILENAME:c9p.db                              # the curve file name
DEGREE                               # introduces the curve degree
2                                                  # curve degree
N.C.P.                 # introduces the Number of Control Points
9                                       # number of control poins
N.KNOTS                           # introduces the Number of Knots
12                                                # number of knots
COORD.C.P.(X,Y,W)         # introduces the CPs coord. and weight
2.775558e-17 3.000000e-01 1.000000e+00            # X Y W values
3.000000e-01 3.000000e-01 7.071070e-01
....
-3.000000e-01 -2.775558e-17 1.000000e+00
-3.000000e-01 3.000000e-01 7.071070e-01
2.775558e-17 3.000000e-01 1.000000e+00
KNOTS                                  # introduces the knot vector
0.000000e+00               # knot values in not decrescent order
0.000000e+00
....
1.000000e+00
1.000000e+00
```

12.2 Interpolation/Approximation points

The following example file is `xcmodel/curves2d/fgo.ip`. The .ip extension identifies a list of points.

```
FILENAME fgo.ip                                    # the file name
N.P. 18         # introduces the Number of Points and the value
-0.030516 -0.373239                       # X Y point coordinates
-0.098592 -0.295775
....
0.105634 -0.316901
0.037559 -0.373239
-0.021127 -0.377934
```

12.3 Control Points

The following example file is `xcmodel/curves2d/spiral.cp`. The .cp extension identifies a list of control points.

```
FILENAME spiral.cp                                 # the file name
N.C.P. 10          # introduces the Number of CPs and the value
0.000000 0.000000 1.000000    # X Y W 2d coordinates and weight
0.000000 -0.100000 1.105168
....
0.400000 -0.300000 7.178994
0.400000 0.400000 13.317328
-0.400000 0.400000 20.908569
-0.400000 -0.500000 26.867794
```

12.4 IGES entity no.126 (NURBS curve 2D)

IGES is the first standard exchange format developed to address the concepts of communicating product data among dissimilar CAD system [ZEID91]. *XCCurv*, being based exclusively on NURBS, implements the entity no.126 only.
The foolowing example file is xcmodel/curves2d/three_circ.igs.

```
                                                                  S      1
1H,,1H;,,14Hthree_circ.igs,                                       G      1
38HXCMODEL ver.3.0 Universita' di Bologna,                        G      2
22Hformato IGES Nov. 2001,32,38,6,308,15,,1.000000E+00,2,2HMM,1,  G      3
1.000000E-02,13H020507.225522,1.000000E-02,3.000000E-01,,,10,0,   G      4
13H020507.225522;                                                 G      5
     126       1       0       1       0       0       0       000000000D      1
     126       0       5      14       0                               0D      2
     126      15       0       1       0       0       0       000000000D      3
```

```
     126       0       5      14       0                                0D      4
     126      29       0       1       0       0       0       000000000D      5
     126       0       5      14       0                                0D      6
126,8,2,0,1,0,0,0.000000E+00,0.000000E+00,0.000000E+00,                  1P      1
2.500000E-01,2.500000E-01,5.000000E-01,5.000000E-01,                     1P      2
7.500000E-01,7.500000E-01,1.000000E+00,1.000000E+00,                     1P      3
1.000000E+00,1.000000E+00,7.071070E-01,1.000000E+00,                     1P      4
7.071070E-01,1.000000E+00,7.071070E-01,1.000000E+00,                     1P      5
7.071070E-01,1.000000E+00,3.038869E-01,2.929329E-01,                     1P      6
0.000000E+00,6.038869E-01,2.929329E-01,0.000000E+00,                     1P      7
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-3.070671E-01,0.000000E+00,3.038869E-01,-3.070671E-01,                   1P      9
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0.000000E+00,0.000000E+00,1.000000E+00,0.000000E+00,                     1P     13
0.000000E+00,1.000000E+00;                                               1P     14
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